Risks of an Intuitive Definition of Control Limits for a Shewhart Control Chart

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Companies maintain quality control charts a thousand times a day in statistical process control (SPC) and use the provided results to gain capable and stable processes. It is of particular importance to check the average location of the main product characteristics for nominal value and tolerance center. You usually use a **Shewhart average chart** for this task. Walter Andrew Shewhart was the first to introduce quality control charts in 1924 and published them in his book "Economic Control of Quality of Manufactured Product" in 1931.

The design of an average chart focuses on a specific question - **how do you choose control limits**?

The upper control limit is referred to as UCL whereas LCL stands for lower control limit. Normal distribution is supposed to be a suitable model in order to describe the monitored characteristic and thus the calculation of the Shewhart average chart is based on this distribution model.

1. Intuitive Definition of Control Limits Relating to the Tolerance

People sometimes use average charts in practice whose control limits are not based on statistical relations. They just select them intuitively, even though they claim these limits to be pragmatic. As an example, they define the control limits in a way that the distance between UCL and LCL amounts to 70% of the tolerance and the two limits are symmetric with respect to the tolerance center.

2. Definition of Control Limits Based on a Statistical Approach

The classical approach is to estimate the parameters of the normal distribution, the expectation μ and the standard deviation σ in the lead time. The accuracy of the estimation depends on the number of units inspected in this lead time. In general, many standards demand a minimum number of 125 units e.g. divided into subsamples of n=5.

The distribution of individuals is known from the normal distribution parameters determined in the lead time. You use the known distribution in order to calculate the expected distribution of averages. And based on the distribution of averages you calculate control limits. The distribution of averages only differs from the distribution of individuals by a slight standard deviation. This leads to:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where

$\sigma_{\bar{x}}$ standard deviation of averages

σ standard deviation of individuals

n sample size of average chart

You define the control limits of the average chart you want to maintain based on the distribution of averages (Figure 1).

It is common practice to use the 99% or 99,73% random variation range for the calculation of control limits. The following example uses the 99,73% random variation range, i.e. the control limits correspond to the $\mu \pm 3 \cdot \sigma_{\bar{x}}$ limits of the distribution of averages. This leads to the following formula for the calculation of the control limits applied in the average chart:

$$\begin{matrix} UCL \\ LCL \end{matrix} = \mu_0 \pm 3 \cdot \frac{\sigma}{\sqrt{n}}$$

You may either replace μ_0 by

- the tolerance center (nominal value) or
- the estimate $\hat{\mu}$ of the expectation from the lead time.

However, the tolerance center is generally preferred. By using the tolerance center, the average chart controls the required nominal value in the long run. In process control, this is more reasonable than controlling an average estimated from the lead time data that is also subject to a certain degree of uncertainty.

For σ you use the estimate $\hat{\sigma}$ calculated in the lead time.

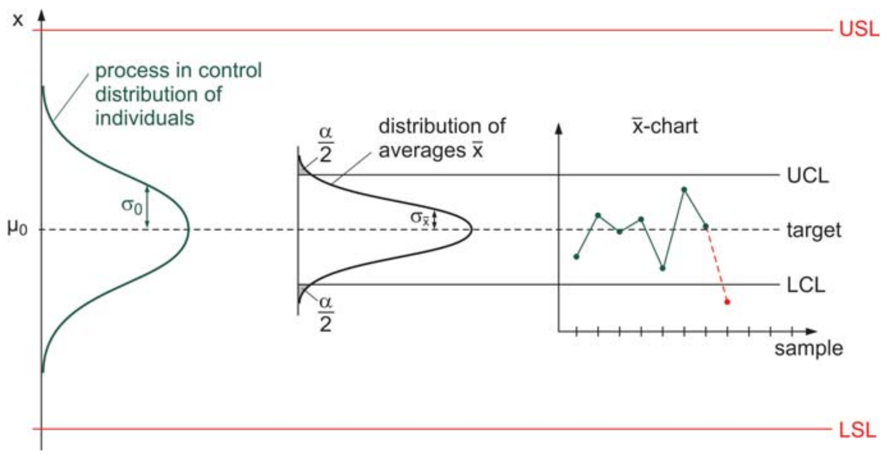

Figure 1: Defining the control limits of the average chart (principle)

3. Comparison

We now compare both approaches for an average chart with the sample size n=5 based on a specific process situation. This situation meets the minimum requirements of most automotive company guidelines. They require a process capability index of at least $C_p=C_{pk}=1{,}33$.

In this case, the standard deviation amounts to $\sigma = \frac{T}{8}$ and the expectation is exactly in the middle of the tolerance center T_m.

Insert these values and n=5 into the calculation formula for the control limits of the Shewhart average chart in order to gain the following equation.

$$\begin{matrix} UCL \\ LCL \end{matrix} = \mu_0 \pm 3 \cdot \frac{\sigma}{\sqrt{n}} = T_m \pm 0{,}1677 \cdot T$$

I.e. the distance between the control limits amounts to about 33,5% of the tolerance.

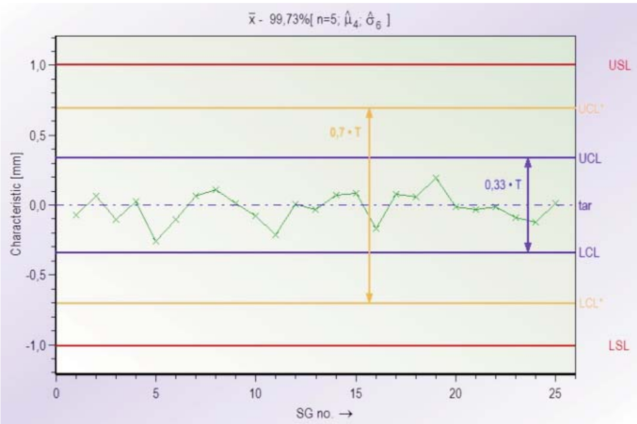

Figure 2: Comparison between the "real" Shewhart average chart and the 70% chart

Figure 2 shows the control limits of the "real" Shewhart average chart (UCL, LCL) and the limits of the 70% chart (UCL*, LCL*) without statistical basis for comparison. In order to understand the context easily, we calculated the charts for a characteristic including the specification of (0±1)mm.

First you notice the considerable difference between the control limits in Figure 2. The distance between the control limits of the intuitive approach is much wider. In addition, you see that the sample size n and the standard deviation σ influence the distance between UCL and LCL considerably in the "real" Shewhart average chart. The intuitive approach completely ignores these two parameters.

In case the process location moves away from the nominal value μ_0, the control chart will indicate this shift in location through averages outside the control limits. How likely is the average chart to detect such a process failure? Figure 3 already shows that this control probability ($1-P_a$) depends on the shift of the expectation.

The graphical display of the control probability subject to the shift in average D_μ compared to the nominal value leads to the operation characteristic. Figure 4 shows the operation characteristics of both charts. The shift in average plotted on the abscissa is standardized with respect to the tolerance.

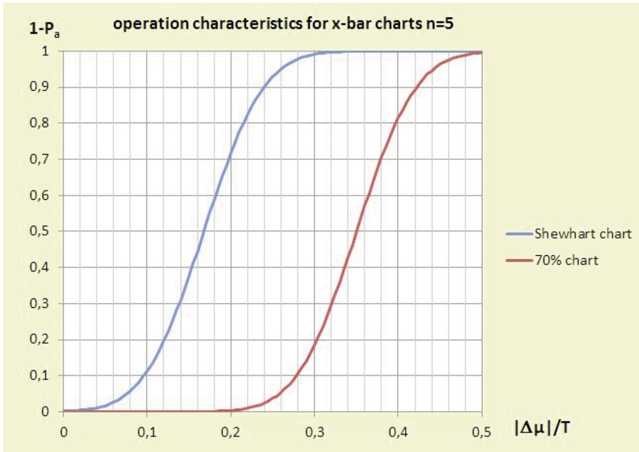

Figure 4: Control line for the Shewhart average chart and the 70% chart in case of n=5

Meaning of the symbols:

$1-P_a$ control probability of the control chart

Δ_μ deviation of the current expectation from the nominal value μ_0 ($\Delta_\mu = |\mu^* - \mu_0|$)

T tolerance

You notice that the 70% chart indicates the shift of the expectation much later than the real Shewhart average chart.

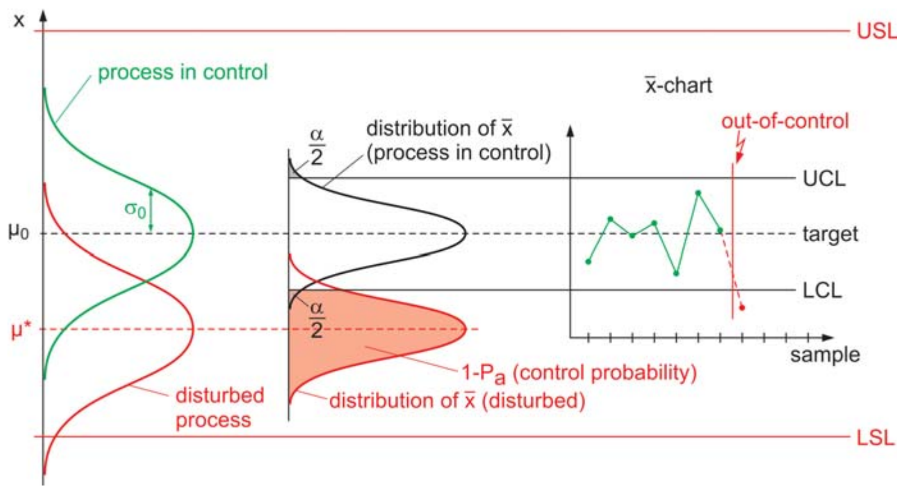

Figure 3: Calculating the control probability of the average chart (principle)

The 70% “control chart“ is not able to indicate relevant changes in the process caused by a sudden or steady shift or trend of the expectation in time. This chart only triggers a possible alarm after the process has already produced a considerable share of rejects!

The following example illustrates this fact. In case the true expectation of our characteristic used in Figure 2 increases from the nominal value of $\mu_0=0$ mm to $\mu^*=0{,}7$ mm, it refers to $\Delta\mu=|\mu^*-\mu_0|=0{,}7$ mm. Referring to the tolerance T=USL-LSL=1-(-1)=2, the deviation amounts to $\Delta\mu/T=0{,}7/2=0{,}35$.

Figure 4 thus shows a control probability of 50% for the 70% chart, i.e. the chance to detect this considerable shift in process location only amounts to 50% when using the 70% chart. The inability of the intuitive 70% chart to detect changes in the process becomes even clearer when calculating the expected share of rejects for the process failure observed.

You gain the result shown in Figure 5 with the help of the qs-STAT functionality *Extras|Probability distribution*. This leads to an expected share of rejects amounting to 11,5%.

Compared to the 70% chart, the control probability of the “real” Shewhart average chart amounts to about 100% for the shift in process location as mentioned before!

Summary

As a result, the calculation of Shewhart average charts should always include the definition of control limits based on statistics as described in section 2.

In case you cannot make any lead time inspections, you may calculate the standard deviation σ from the minimum requirement of C_p in exceptional cases:

$$\sigma = \frac{T}{6 \cdot C_p}$$

However, please consider that the true standard deviation of the process may differ from the calculated deviation. For this reason, it makes sense to determine the standard deviation empirically after recording a certain number of values in the control chart. Now you may even calculate control limits again.

If possible, try to avoid control limits for average charts that are defined intuitively based on certain %-regions of the tolerance. Otherwise, it is advisable to at least challenge your limits in order to find out whether they are able to detect process failures.

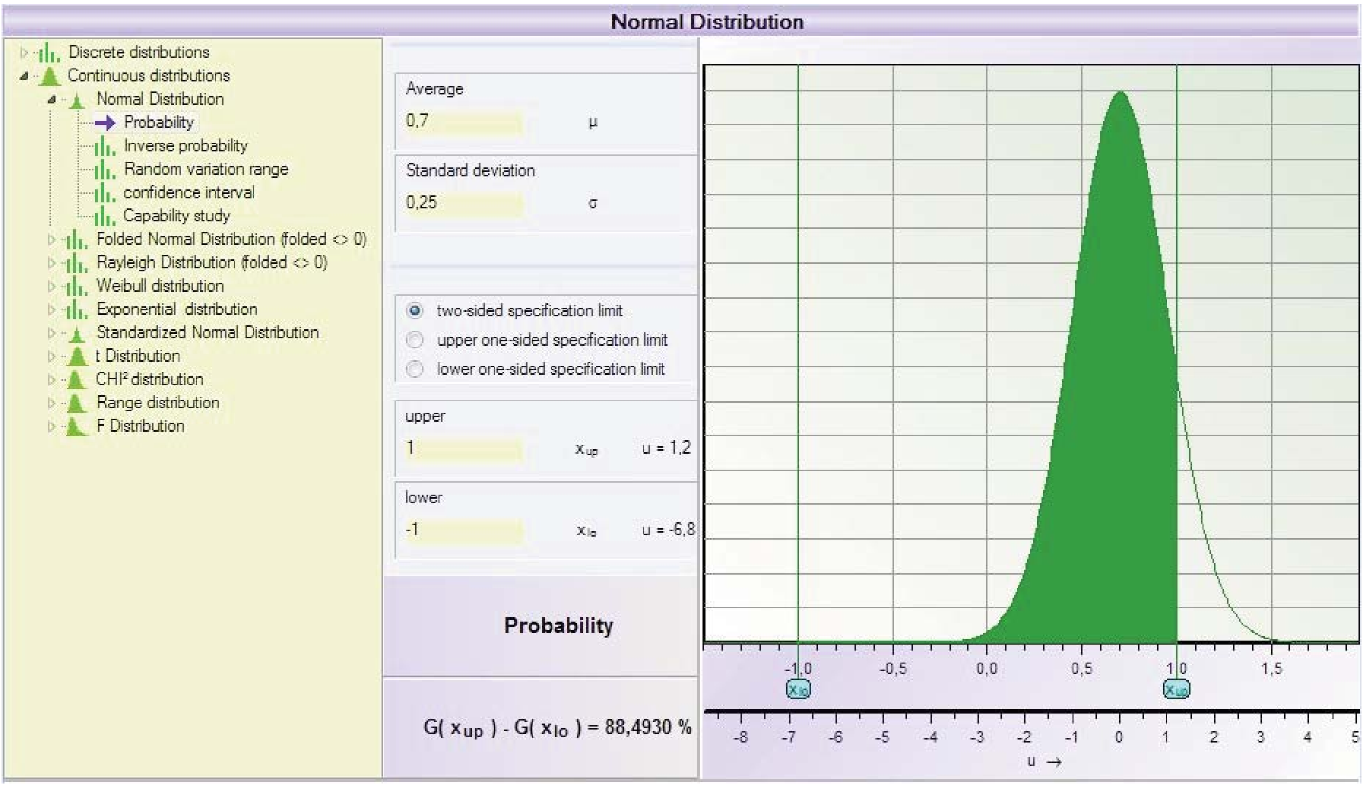

Figure 5: Yield and share of rejects in case of a shift in location of Δμ/T=0,35